15

$$U_{p2} = U_{p3}$$
 (18)

From Eq. (4)

$$\begin{split} & \mathbf{P}_{1} - \mathbf{P}_{0} = \rho_{0} (\mathbf{U}_{\text{s}1} - \mathbf{U}_{\text{p}0}) (\mathbf{U}_{\text{p}1} - \mathbf{U}_{\text{p}0}), \\ & \mathbf{P}_{2} - \mathbf{P}_{0} = \rho_{0}' (\mathbf{U}_{\text{s}2} - \mathbf{U}_{\text{p}0}) (\mathbf{U}_{\text{p}2} - \mathbf{U}_{\text{p}0}), \\ & \mathbf{P}_{3} - \mathbf{P}_{1} = \rho_{1} (\mathbf{U}_{\text{s}3} - \mathbf{U}_{\text{p}1}) (\mathbf{U}_{\text{p}3} - \mathbf{U}_{\text{p}1}). \end{split}$$

The material ahead of the shock waves  $U_{s1}$  and  $U_{s2}$  is assumed at rest so that  $P_0 = U_{p0} = 0$ . Using the continuity conditions and the first and third of the above equations,

$$P_{2} = \rho_{0}U_{s1}U_{p1} + \rho_{1}(U_{s3}-U_{p1})(U_{p2}-U_{p1}).$$

Substituting for P2 and simplifying

$$\frac{U_{p2}}{U_{p1}} = \frac{\rho_0 U_{s1} - \rho_1 (U_{s3} - U_{p1})}{\rho_0 U_{s2} - \rho_1 (U_{s3} - U_{p1})}$$
(19)

and

$$\frac{P_2}{P_1} = \frac{\rho'_0 U_{s2} \left[\rho_0 U_{s1} - \rho_1 (U_{s3} - U_{p1})\right]}{\rho_0 U_{s3} \left[\rho'_0 U_{s2} - \rho_1 (U_{s3} - U_{p1})\right]}$$
(20)

The shock waves moving to the right, such as  $U_{s1}$  and  $U_{s2}$  are defined as traveling in the positive direction and those to the left, such as  $U_{s3}$  are moving in the negative direction. In an experiment, the impedances  $\rho_0 U_{s1}$  and  $\rho'_0 U_{s2}$  for the media are readily determined from the measured shock velocities. However, the quantity  $\rho_1 U_{s3}$  is very troublesome due to the difficulty in measuring  $U_{s3}$  by the technique used here. This problem can be overcome if an extension of the acoustic approximation<sup>19</sup> is used to write

$$\rho_0 U_{s1} = -\rho_1 (U_{s3} - U_{p1}). \tag{21}$$

Equations (19) and (20) can then be simplified to read